MEASUREMENT OF THE ELECTROMAGNETIC PROPERTIES OF BIAXIAL ANISOTROPIC MATERIALS USING A WAVEGUIDE TECHNIQUE

Benjamin Reid Crowgey, Raoul Ouedraogo, Edward Rothwell, Leo Kempel, and Shanker Balasubramaniam

Department of Electrical and Computer Engineering, Michigan State University

2010 Inverse Problems Symposium
June 7, 2010
Kellogg Center

1. Overview
   - Material Characterization
   - Goal
2. Approach
   - Biaxial Waveguide Technique Theory
   - Measurement Procedure
3. Results
   - Monte Carlo Error Analysis
4. Conclusion

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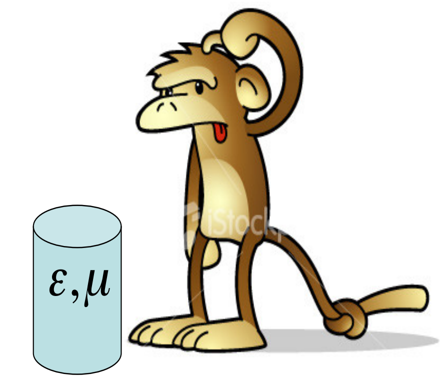
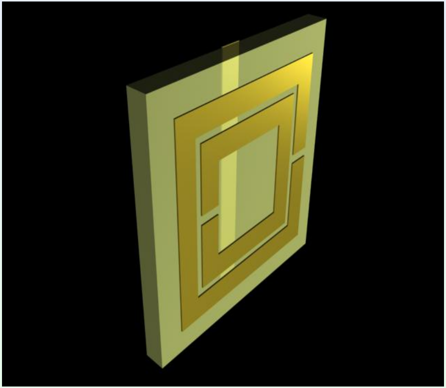
Electromagnetic Properties of Materials



Permittivity and Permeability

- Accurate knowledge required
- Verification of manufactured materials
- Properties of newly introduced materials
- Measurement of simple homogeneous, isotropic, linear materials remains a challenge

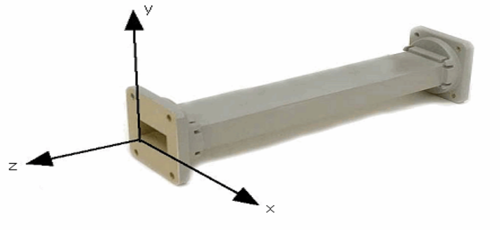
Ainsotropic Materials



Permittivity and Permeability

- Directionally dependent
- Interests in material characterization raised due to introduction of metamaterials
  - Manufactured materials with EM properties not found in nature
- Metamaterial properties only determined from measurement

Waveguide Technique



Isotropic Material

- Expose a sample of material to an EM wave
  - Two parameters (ϵ,μ) require two independent measurements
  - $\longrightarrow$ reflection and transmission of wave
- Rectangular Waveguide
  - Confinement of wave produces good signal strength

Waveguide Technique

Guided Wave Theory

- Closed-form inverse expressions for permittivity and permeability obtained
- Well-conditioned for most material parameters of interest
  - Except when material thickness approaches multiple of half wavelength
  - Subject to high error but is easily understood, predicted and compensated for by control of sample thickness
- Riemann-sheet ambiguity introduced in closed-form expression through complex logarithm

Waveguide Technique

$$[\epsilon] = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

$$[\mu] = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

Biaxial Anisotropic Material

- Material responds differently depending on polarization of incident wave
- No coupling between orthogonal field components
- Six complex material parameters
  - Three different sets of transmission and reflection measurements required
- All six closed form solutions available

Goal

Biaxial Waveguide Technique

- Describe waveguide technique for biaxial materials
- Provide results using extraction method
- Explore parameter sensitivity by performing a Monte Carlo error analysis

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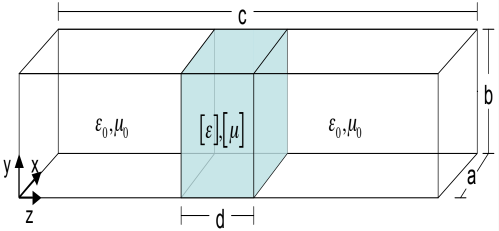
TE_{n0} Wave Propagation



Wave Equation

- Waveguide partially filled with biaxial material

$$\left(\frac{\partial^2}{\partial x^2} + k_c^2\right) H_z = 0$$

$$k_c = \frac{\mu_z}{\mu_x}\left(\omega^2 \mu_x \epsilon_y - \beta^2\right)$$

TE_{n0} Wave Propagation

Field Structure

- TE_{10} biaxial-filled waveguide mode will couple into TE_{10} empty waveguide mode since field structure is identical

$$H_z(x,z) = B\cos\left(\frac{\pi}{a}x\right)e^{\pm j\beta z}$$

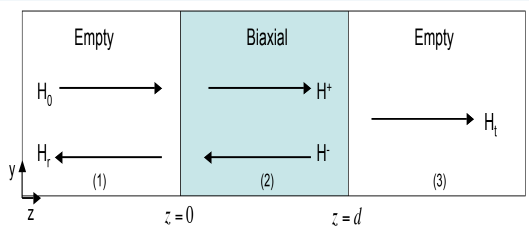
- Cutoff wavenumber for TE_{10} mode gives propagation constant β

$$k_c = \frac{\pi}{a} = \frac{\mu_z}{\mu_x}\left(\omega^2\mu_x\epsilon_y - \beta^2\right)$$

- Interfacial reflection coefficient and propagation factor

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}, \quad P = e^{-j\beta d}$$

TE$_{n0}$ Wave Propagation



Complex Wavenumber

- Regions 1 and 3

$$\beta_0 = \sqrt{k_0^2 - k_c^2}$$

- Region 2

$$\beta = \sqrt{\omega^2 \mu_x \epsilon_y - \frac{\mu_x}{\mu_z} k_c^2}$$

Nicolson-Ross Technique

S-Parameters

- Using this technique certain material parameters are extracted from measured S-Parameters

$$V_1 = S_{21} + S_{11}$$

$$V_2 = S_{21} - S_{11}$$

- The reflection coefficient between faces of biaxial material and empty waveguide is found in terms of measured S-Parameter

$$\Gamma = \frac{1 - V_1 V_2}{V_1 - V_2} \pm \sqrt{\left(\frac{1 - V_1 V_2}{V_1 - V_2}\right)^2 - 1}$$

Nicolson-Ross Technique

S-Parameters

- The propagation factor in turn can be derived in terms of the S-parameter and Γ

$$P = \frac{V_1 - \Gamma}{1 - \Gamma V_1}$$

- After obtaining Γ and P, β is computed by

$$\beta = \frac{\ln P \pm j2n\pi}{-jd}$$

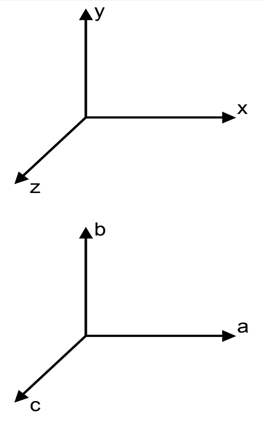
  - Ambiguity (choice of n) introduced in the closed-form expression through this complex logarithm

Measurement Procedure

$$[\epsilon] = \begin{bmatrix} \epsilon_a & 0 & 0 \\ 0 & \epsilon_b & 0 \\ 0 & 0 & \epsilon_c \end{bmatrix}$$

$$[\mu] = \begin{bmatrix} \mu_a & 0 & 0 \\ 0 & \mu_b & 0 \\ 0 & 0 & \mu_c \end{bmatrix}$$

Measurement Procedure



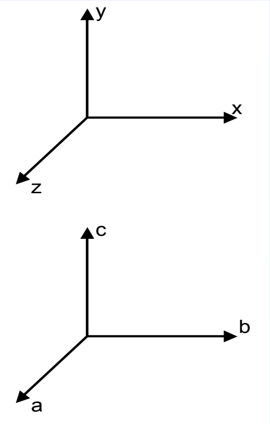
Measurement 1

- Line up so $\mu_a = \mu_x$, $\mu_b = \mu_y$, and $\mu_c = \mu_z$

$$\frac{\mu_a}{\mu_0} = \frac{1+\Gamma}{1-\Gamma}\frac{\beta}{\beta_0}$$

$$\frac{\epsilon_b}{\epsilon_0} = \frac{\beta\beta_0\left(1-\Gamma\right)}{k_0^2\left(1+\Gamma\right)} + \frac{\mu_0}{\mu_c}\left(\frac{k_c}{\beta_0}^2\right)$$

Measurement Procedure



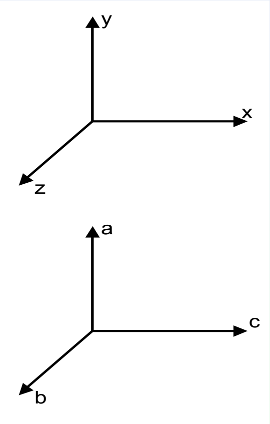
Measurement 2

- Line up so $\mu_b = \mu_x$, $\mu_c = \mu_y$, and $\mu_a = \mu_z$

$$\frac{\mu_b}{\mu_0} = \frac{1+\Gamma}{1-\Gamma}\frac{\beta}{\beta_0}$$

$$\frac{\epsilon_c}{\epsilon_0} = \frac{\beta\beta_0\left(1-\Gamma\right)}{k_0^2\left(1+\Gamma\right)} + \frac{\mu_0}{\mu_a}\left(\frac{k_c}{\beta_0}^2\right)$$

Measurement Procedure



Measurement 3

- Line up so $\mu_c = \mu_x$, $\mu_a = \mu_y$, and $\mu_b = \mu_z$

$$\frac{\mu_c}{\mu_0} = \frac{1+\Gamma}{1-\Gamma}\frac{\beta}{\beta_0}$$

$$\frac{\epsilon_a}{\epsilon_0} = \frac{\beta\beta_0\left(1-\Gamma\right)}{k_0^2\left(1+\Gamma\right)} + \frac{\mu_0}{\mu_b}\left(\frac{k_c}{\beta_0}^2\right)$$

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Error Analysis Setup

$$[\epsilon] = \epsilon_0 \begin{bmatrix} 2.0 & 0 & 0 \\ 0 & 2.35 & 0 \\ 0 & 0 & 3.50 \end{bmatrix}$$

$$[\mu] = \mu_0 \begin{bmatrix} 2.75 & 0 & 0 \\ 0 & 2.25 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Parameters

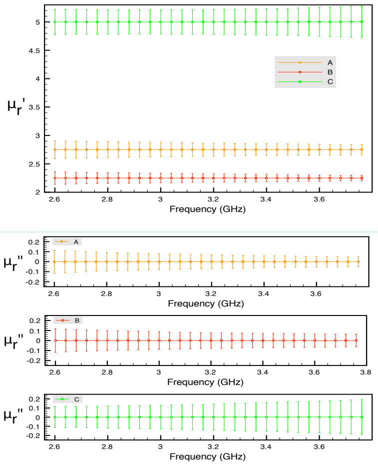
- S Band waveguide
- Frequency goes from 2.6 GHz to 3.8 GHz
- Thickness of material is 10 mm for each sample

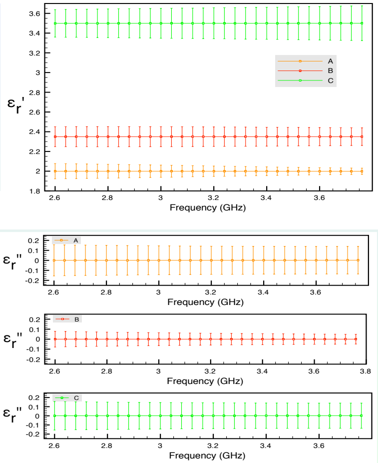
Error in S-Parameter Measurements

S-Parameters

- $S_{11} = \frac{V^{Refl}}{V^{Inc}}$ and $S_{21} = \frac{V^{Trans}}{V^{Inc}}$
- S-Parameters measured on Network Analzser
- $\sigma_{|S_{11}|} = 0.004$ and $\sigma_{\angle S_{11}} = 0.8^\circ$
- $\sigma_{|S_{21}|} = 0.16$ and $\sigma_{\angle S_{21}} = 2.0^\circ$
- 100,000 monte carlo trials
- Error bars show the 95% (2σ) confidence level due to network analyzer uncertainty for an HP8510 network analyzer

Error in S-Parameter Measurements



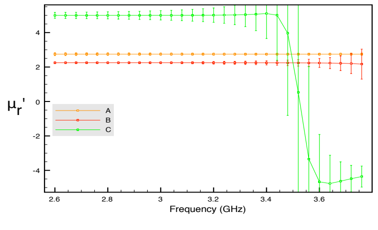


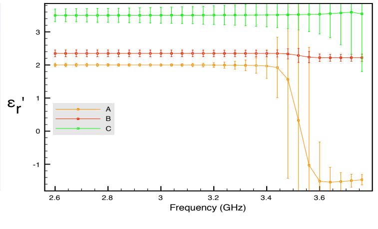
Error in S-Parameter Measurements

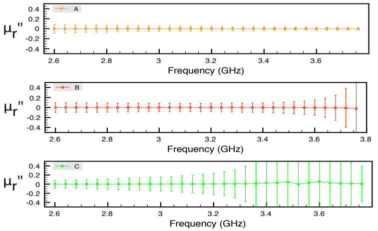
Ambiguity from Complex Logarithm

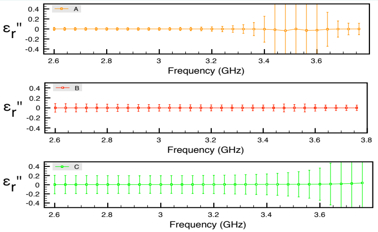
- $\beta = \frac{\ln P \pm j2n\pi}{-jd}$
- Choice of n error from Riemann-sheet ambiguity in closed-form expression of β
- Depends on thickness of material under test in comparison to wavelength of propagating field
- When $d < \frac{\lambda}{2}$, n is 0 and increments by 1 every λ in thickness
- Increased thickness to 14 mm

Error in S-Parameter Measurements with Thicker Sample









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Conclusion

- Biaxial anisotropic material extraction Method
- Error analysis on sensitivity of S-parameters
- Consequences of choosing wrong of n

Future Work

Further Study

- Error sensitivity on the thickness of the sample
- Error sensitivity on position of where the S-Parameters are measured in the waveguide
- Solving the n ambiguity
- Investigating samples with complex material parameters